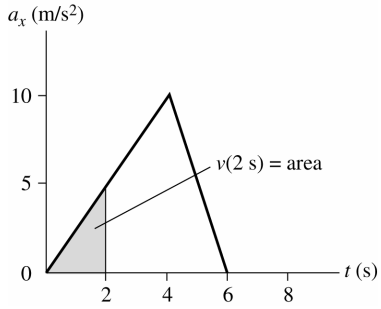
2.28. Visualize:



Solve: We will determine the object's velocity using graphical methods first and then using calculus. Graphically, $v(t) = v_0 +$ area under the acceleration curve from 0 to t. In this case, $v_0 = 0$ m/s. The area at each time t requested is a triangle.

$$t = 0 \text{ s} \qquad v(t = 0 \text{ s}) = v_0 = 0 \text{ m/s}$$

$$t = 2 \text{ s} \qquad v(t = 2 \text{ s}) = \frac{1}{2}(2 \text{ s})(5 \text{ m/s}) = 5 \text{ m/s}$$

$$t = 4 \text{ s} \qquad v(t = 4 \text{ s}) = \frac{1}{2}(4 \text{ s})(10 \text{ m/s}) = 20 \text{ m/s}$$

$$t = 6 \text{ s} \qquad v(t = 6 \text{ s}) = \frac{1}{2}(6 \text{ s})(10 \text{ m/s}) = 30 \text{ m/s}$$

$$t = 8 \text{ s} \qquad v(t = 8 \text{ s}) = v(t = 6 \text{ s}) = 30 \text{ m/s}$$

The last result arises because there is no additional area after $t = 6$ s. Let us now use calculus. The acceleration function $a(t)$ consists of three pieces and can be written:

$$a(t) = \begin{cases} 2.5t & 0 \text{ s} \le t \le 4 \text{ s} \\ -5t + 30 & 4 \text{ s} \le t \le 6 \text{ s} \\ 0 & 6 \text{ s} \le t \le 8 \text{ s} \end{cases}$$

These were determined by the slope and the y-intercept of each of the segments of the graph. The velocity function is found by integration as follows: For $0 \le t \le 4$ s,

$$v(t) = v(t = 0 \text{ s}) + \int_0^t a(t)dt = 0 + 2.5\frac{t^2}{2}\Big|_0^t = 1.25t^2$$

This gives

$$t = 0 \text{ s} \qquad v(t = 0 \text{ s}) = 0 \text{ m/s}$$

$$t = 2 \text{ s} \qquad v(t = 2 \text{ s}) = 5 \text{ m/s}$$

$$t = 4 \text{ s} \qquad v(t = 4 \text{ s}) = 20 \text{ m/s}$$

For $4 \text{ s} \le t \le 6$ s,

$$v(t) = v(t = 4 \text{ s}) + \int_4^t a(t)dt = 20 \text{ m/s} + \left[\frac{-5t^2}{2} + 30t\right]_4^t = -2.5t^2 + 30t - 60$$

This gives:

$$t = 6 \text{ s} \qquad v(t = 6 \text{ s}) = 30 \text{ m/s}$$

For $6 \text{ s} \le t \le 8$ s,

$$v(t) = v(t = 6 \text{ s}) + \int_6^t a(t)dt = 30 \text{ m/s} + 0 \text{ m/s} = 30 \text{ m/s}$$

This gives:

$$t = 8 \text{ s} \qquad v(t = 8 \text{ s}) = 30 \text{ m/s}$$

Assess: The same velocities are found using calculus and graphs, but the graphical method is easier for simple graphs.